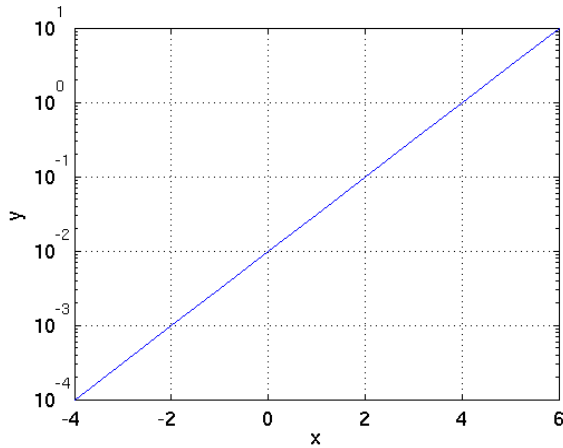


Example final exam, Math 445, Intro to Numerical Mathematics with Matlab **Name:**

Note: This example final exam is meant to give you an idea of the scope and length of questions on the final exam. But the questions on the actual final exam will probably be quite different.

Problem 1. Write Matlab code for plotting $y = 6.2x^{-4}$ as a black line, for 30 evenly spaced points in x between 0.01 and 10. Choose a plotting function that highlights the functional relationship between y and x . Label the axes and title the graph.

Problem 2. Write an equation for y as a function of x for the following data plot.



Problem 3. Write Matlab code that produces a contour plot of the function

$$f(x, y) = \frac{\sin \sqrt{x^2 + y^2}}{x^2 + y^2}$$

over the rectangle $-10 \leq x \leq 10$ and $-5 \leq y \leq 5$, with 30 evenly spaced points in each of x and y . Label the axes.

Problem 4. Write Matlab code that produces the solution of the following system of equations

$$\begin{aligned}x + y + z &= 3 \\2x + 3w - 4z &= -5 \\4w - 3x + 2y + z &= 8 \\-w + 7x + 6y &= 7\end{aligned}$$

Problem 5. Write one line of Matlab code that would verify that the solution computed in the previous problem satisfies the given equations.

Problem 6. Write a Matlab file function that returns a numerical approximation to $\cos x$ using the following series, where x and N are input arguments to the function. You can use Matlab's `factorial` function to compute the factorial.

$$\cos x \approx \sum_{n=0}^N (-1)^n \frac{x^{2n}}{(2n)!}$$

Problem 7. The Rössler system is a ordinary differential equation in three variables $[x, y, z]$ defined by

$$\frac{d}{dt} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{f} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y - z \\ x + ay \\ b + z(x - c) \end{pmatrix}$$

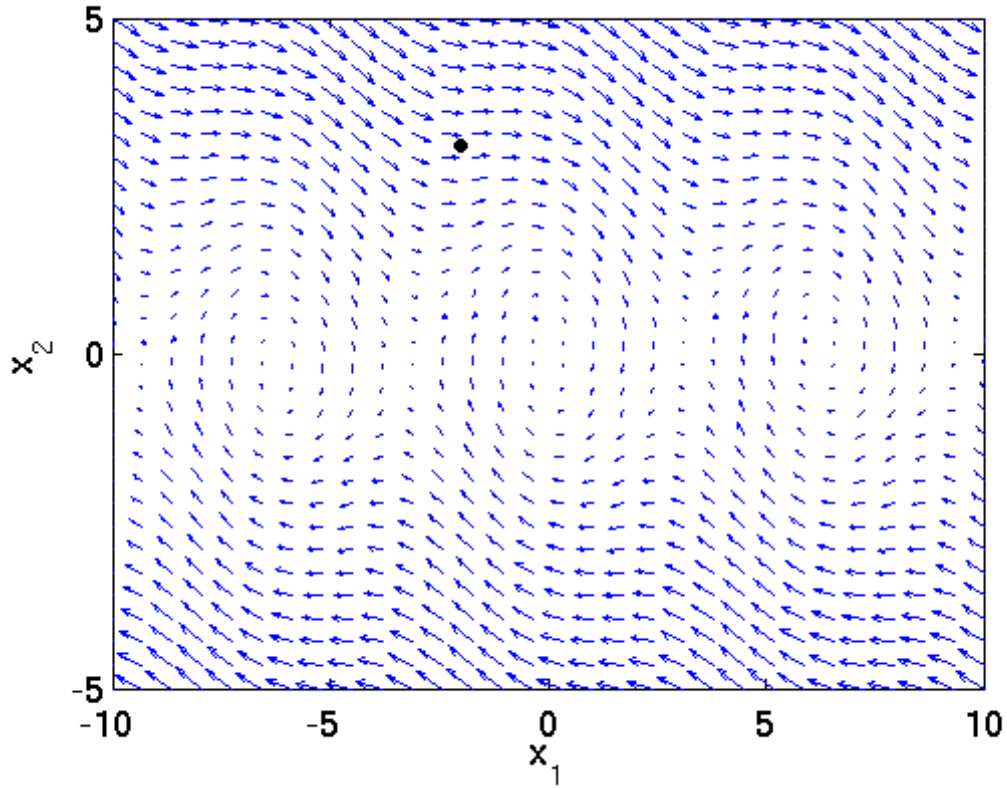
Let the vector \mathbf{x} be the column vector $(x, y, z)'$. Write Matlab code for an anonymous function that returns the vector $\mathbf{f}(\mathbf{x})$ given an input vector \mathbf{x} for the parameter values $a = 0.1$, $b = 0.1$, and $c = 14$.

Problem 8. For the nonlinear function $\mathbf{f}(\mathbf{x})$ given in problem 7, the equation $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ has a solution near the point $(x, y, z) = (14, -140, 140)$. Given your answer to problem 7, write down the Matlab code that would find the precise value of (x, y, z) near this point for which $\mathbf{f}(\mathbf{x}) = \mathbf{0}$.

Problem 9. Revise your answer to problem 7 so that the anonymous function is in the right form to be passed into Matlab's `ode45` function, and write down the Matlab code that would integrate the Rössler system from $t = 0$ to $t = 100$, from the initial condition $x = 0, y = 1, z = 0$.

Problem 10. The following quiver plot shows the vector field of the differential equation $dx/dt = f(x)$.

(a) Sketch on the quiver plot the solution $\mathbf{x}(t)$ of the differential equation that starts from the initial condition $\mathbf{x}(0) = (x_1, x_2) = (-2, 3)$ (the solid dot on the figure).



(b) What physical system does this differential equation model? The answer has four words.