IAM 950: Spatiotemporal dynamics and turbulence

New course proposal. J. Gibson and G. Chini, U. New Hampshire, Oct. 12 2011

Course abstract: This course examines the theory of bifurcations, pattern formation, and spatiotemporal chaos in systems governed by time-dependent nonlinear partial differential equations. The course begins with comparatively well-developed theories and simple model systems (e.g. 1-dimensional nonlinear PDEs) and progresses by stages of increasing complexity to the culminating problem of coherent structures and turbulence in the Navier-Stokes equations. The goal is to give advanced graduate students the proper analytic and conceptual background for research in the dynamics of unstable spatio-temporal patterns in turbulent shear flows.

Instructors:

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Primary text:

Pattern Formation and Dynamics in Nonequilibrium Systems, Michael Cross and Henry Greenside, Cambridge University Press, 2009.

Supplementary texts:

Dissipative Structure & Weak Turbulence, Paul Manneville, Academic Press, 1990.

Instabilities, Chaos, and Turbulence, Paul Manneville, Imperial College Press, 2010.

Pattern Formation: An Introduction to Methods, Rebecca Hoyle, Cambridge Texts in Applied Mathematics, 2006.

Stability and Transition in Shear Flows, Peter Schmid and Dan Henningson, Springer-Verlag, 2000.

Turbulence, Coherent Structures, Dynamical Systems, and Symmetry, Philip Holmes, John Lumley, and Gal Berkooz, Cambridge University Press, 1996

Chaos: Classical and Quantum, Predrag Cvitanović, www.chaosbook.org, 2011.

Linear and Nonlinear Waves, Gerald Whitham, Wiley & Sons, 1973

Evaluation: Seven biweekly homework assignments and class participation.

Syllabus:

Introduction (two weeks)

pattern formation in forced dissipative spatial systems
linear stability and bifurcation in spatial systems, Types I, II, III
numerical simulation methods
examples: 1d & 2d Swift-Hohenberg, Rayleigh-Bénard convection

Weakly nonlinear spatial systems (three weeks)

nonlinear saturation and mode reduction
amplitude equations (1d for type I instabilities)
applications of amplitude equations
examples: 1d & 2d Swift-Hohenberg, 1d Kuramoto-Sivashinksy

Strongly nonlinear spatial systems (three weeks)

linear WKB theory (oscillators and waves)
weakly nonlinear WKB theory (dispersive waves)
strongly nonlinear WKB theory (complex Ginzburg-Landau)
phase diffusion/modulation theory, stability balloons nonlinear WKB analysis

Unstable spatio-temporal patterns in turbulent flows (three weeks)

canonical shear flows and the Navier-Stokes equations
linear stability of laminar solutions
instabilities of slowly decaying laminar eigenfunctions
far-from-laminar unstable equilibria, traveling waves
spatially localized solutions
recurrent unstable temporal patterns (periodic orbits)

Periodic orbit theory (three weeks)

motivation in low-d dynamical systems
approximating invariant measures of chaotic systems
Perron-Frobenius operators
trace formulae, spectral determinants
derivation of cycle expansion formulae